



## **AP<sup>®</sup> Calculus AB 2011 Scoring Guidelines**

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**Question 1**

For  $0 \leq t \leq 6$ , a particle is moving along the  $x$ -axis. The particle's position,  $x(t)$ , is not explicitly given. The velocity of the particle is given by  $v(t) = 2\sin(e^{t/4}) + 1$ . The acceleration of the particle is given by  $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$  and  $x(0) = 2$ .

- (a) Is the speed of the particle increasing or decreasing at time  $t = 5.5$ ? Give a reason for your answer.  
 (b) Find the average velocity of the particle for the time period  $0 \leq t \leq 6$ .  
 (c) Find the total distance traveled by the particle from time  $t = 0$  to  $t = 6$ .  
 (d) For  $0 \leq t \leq 6$ , the particle changes direction exactly once. Find the position of the particle at that time.

(a)  $v(5.5) = -0.45337$ ,  $a(5.5) = -1.35851$

The speed is increasing at time  $t = 5.5$ , because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity  $= \frac{1}{6} \int_0^6 v(t) dt = 1.949$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Distance  $= \int_0^6 |v(t)| dt = 12.573$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d)  $v(t) = 0$  when  $t = 5.19552$ . Let  $b = 5.19552$ .  
 $v(t)$  changes sign from positive to negative at time  $t = b$ .  
 $x(b) = 2 + \int_0^b v(t) dt = 14.134$  or  $14.135$

3 :  $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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**Question 2**

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

(a) 
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

(b)  $\frac{1}{10} \int_0^{10} H(t) dt$  is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left( 2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

(c)  $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$   
The temperature of the tea drops 23 degrees Celsius from time  $t = 0$  to time  $t = 10$  minutes.

(d)  $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$ ;  $H(10) - B(10) = 8.817$   
The biscuits are 8.817 degrees Celsius cooler than the tea.

1 : answer

3 :  $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

2 :  $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

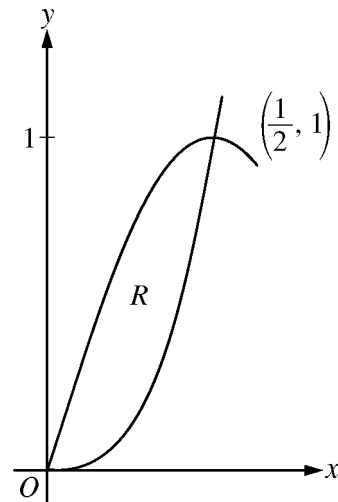
3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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**Question 3**

Let  $R$  be the region in the first quadrant enclosed by the graphs of  $f(x) = 8x^3$  and  $g(x) = \sin(\pi x)$ , as shown in the figure above.

- (a) Write an equation for the line tangent to the graph of  $f$  at  $x = \frac{1}{2}$ .
- (b) Find the area of  $R$ .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .



(a)  $f\left(\frac{1}{2}\right) = 1$

$$f'(x) = 24x^2, \text{ so } f'\left(\frac{1}{2}\right) = 6$$

An equation for the tangent line is  $y = 1 + 6\left(x - \frac{1}{2}\right)$ .

(b) Area =  $\int_0^{1/2} (g(x) - f(x)) dx$

$$= \int_0^{1/2} (\sin(\pi x) - 8x^3) dx$$

$$= \left[ -\frac{1}{\pi} \cos(\pi x) - 2x^4 \right]_{x=0}^{x=1/2}$$

$$= -\frac{1}{8} + \frac{1}{\pi}$$

(c)  $\pi \int_0^{1/2} ((1 - f(x))^2 - (1 - g(x))^2) dx$

$$= \pi \int_0^{1/2} ((1 - 8x^3)^2 - (1 - \sin(\pi x))^2) dx$$

$$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$$

$$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$$

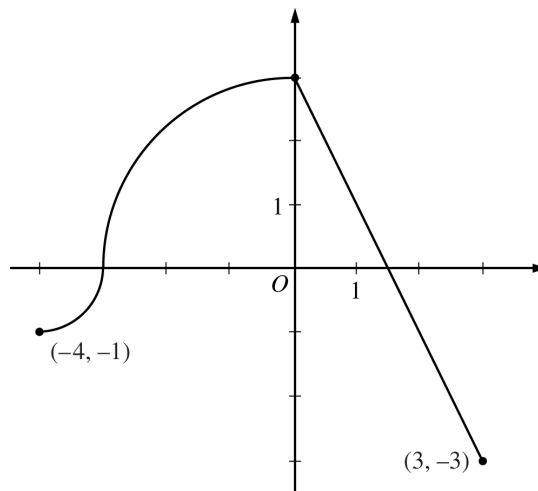
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**Question 4**

The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above.

Let  $g(x) = 2x + \int_0^x f(t) dt$ .

- (a) Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
- (b) Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
- (c) Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of  $f$

(a)  $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$   
 $g'(x) = 2 + f(x)$   
 $g'(-3) = 2 + f(-3) = 2$

$$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$$

(b)  $g'(x) = 0$  when  $f(x) = -2$ . This occurs at  $x = \frac{5}{2}$ .  
 $g'(x) > 0$  for  $-4 < x < \frac{5}{2}$  and  $g'(x) < 0$  for  $\frac{5}{2} < x < 3$ .  
 Therefore  $g$  has an absolute maximum at  $x = \frac{5}{2}$ .

$$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$$

(c)  $g''(x) = f'(x)$  changes sign only at  $x = 0$ . Thus the graph of  $g$  has a point of inflection at  $x = 0$ .

1 : answer with reason

(d) The average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$  is  
 $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$ .

$$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$$

To apply the Mean Value Theorem,  $f$  must be differentiable at each point in the interval  $-4 < x < 3$ . However,  $f$  is not differentiable at  $x = -3$  and  $x = 0$ .

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**Question 5**

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function  $W$  models the total amount of solid waste stored at the landfill. Planners estimate that  $W$  will satisfy the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  for the next 20 years.  $W$  is measured in tons, and  $t$  is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of  $W$  at  $t = 0$  to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time  $t = \frac{1}{4}$ ).
- (b) Find  $\frac{d^2W}{dt^2}$  in terms of  $W$ . Use  $\frac{d^2W}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time  $t = \frac{1}{4}$ .
- (c) Find the particular solution  $W = W(t)$  to the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  with initial condition  $W(0) = 1400$ .

(a)  $\left. \frac{dW}{dt} \right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is  $y = 1400 + 44t$ .

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411 \text{ tons}$$

(b)  $\frac{d^2W}{dt^2} = \frac{1}{25} \frac{dW}{dt} = \frac{1}{625}(W - 300)$  and  $W \geq 1400$

Therefore  $\frac{d^2W}{dt^2} > 0$  on the interval  $0 \leq t \leq \frac{1}{4}$ .

The answer in part (a) is an underestimate.

(c)  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$$\int \frac{1}{W - 300} dW = \int \frac{1}{25} dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$$

$$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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**Question 6**

Let  $f$  be a function defined by  $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that  $f$  is continuous at  $x = 0$ .  
 (b) For  $x \neq 0$ , express  $f'(x)$  as a piecewise-defined function. Find the value of  $x$  for which  $f'(x) = -3$ .  
 (c) Find the average value of  $f$  on the interval  $[-1, 1]$ .

(a)  $\lim_{x \rightarrow 0^-} (1 - 2\sin x) = 1$

$$\lim_{x \rightarrow 0^+} e^{-4x} = 1$$

$$f(0) = 1$$

So,  $\lim_{x \rightarrow 0} f(x) = f(0)$ .

Therefore  $f$  is continuous at  $x = 0$ .

(b)  $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$$-2\cos x \neq -3 \text{ for all values of } x < 0.$$

$$-4e^{-4x} = -3 \text{ when } x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0.$$

Therefore  $f'(x) = -3$  for  $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$ .

(c) 
$$\begin{aligned} \int_{-1}^1 f(x) dx &= \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx \\ &= \int_{-1}^0 (1 - 2\sin x) dx + \int_0^1 e^{-4x} dx \\ &= \left[ x + 2\cos x \right]_{x=-1}^{x=0} + \left[ -\frac{1}{4}e^{-4x} \right]_{x=0}^{x=1} \\ &= (3 - 2\cos(-1)) + \left( -\frac{1}{4}e^{-4} + \frac{1}{4} \right) \end{aligned}$$

$$\begin{aligned} \text{Average value} &= \frac{1}{2} \int_{-1}^1 f(x) dx \\ &= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4} \end{aligned}$$

2 : analysis

3 :  $\begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

4 :  $\begin{cases} 1 : \int_{-1}^0 (1 - 2\sin x) dx \text{ and } \int_0^1 e^{-4x} dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$