



## **AP<sup>®</sup> Calculus BC 2011 Scoring Guidelines**

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**Question 1**

At time  $t$ , a particle moving in the  $xy$ -plane is at position  $(x(t), y(t))$ , where  $x(t)$  and  $y(t)$  are not explicitly given. For  $t \geq 0$ ,  $\frac{dx}{dt} = 4t + 1$  and  $\frac{dy}{dt} = \sin(t^2)$ . At time  $t = 0$ ,  $x(0) = 0$  and  $y(0) = -4$ .

- (a) Find the speed of the particle at time  $t = 3$ , and find the acceleration vector of the particle at time  $t = 3$ .  
 (b) Find the slope of the line tangent to the path of the particle at time  $t = 3$ .  
 (c) Find the position of the particle at time  $t = 3$ .  
 (d) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .

(a) Speed =  $\sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$  or  $13.007$

Acceleration =  $\langle x''(3), y''(3) \rangle$   
 $= \langle 4, -5.466 \rangle$  or  $\langle 4, -5.467 \rangle$

2 :  $\left\{ \begin{array}{l} 1 : \text{speed} \\ 1 : \text{acceleration} \end{array} \right.$

(b) Slope =  $\frac{y'(3)}{x'(3)} = 0.031$  or  $0.032$

1 : answer

(c)  $x(3) = 0 + \int_0^3 \frac{dx}{dt} dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt} dt = -3.226$

At time  $t = 3$ , the particle is at position  $(21, -3.226)$ .

4 :  $\left\{ \begin{array}{l} 2 : x\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : y\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(d) Distance =  $\int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 21.091$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

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**Question 2**

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

(a) 
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

(b)  $\frac{1}{10} \int_0^{10} H(t) dt$  is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left( 2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

(c)  $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$   
 The temperature of the tea drops 23 degrees Celsius from time  $t = 0$  to time  $t = 10$  minutes.

(d)  $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$ ;  $H(10) - B(10) = 8.817$   
 The biscuits are 8.817 degrees Celsius cooler than the tea.

1 : answer

3 :  $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

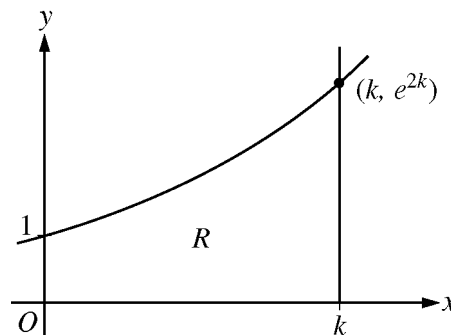
2 :  $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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**Question 3**

Let  $f(x) = e^{2x}$ . Let  $R$  be the region in the first quadrant bounded by the graph of  $f$ , the coordinate axes, and the vertical line  $x = k$ , where  $k > 0$ . The region  $R$  is shown in the figure above.



- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of  $R$  in terms of  $k$ .
- (b) The region  $R$  is rotated about the  $x$ -axis to form a solid. Find the volume,  $V$ , of the solid in terms of  $k$ .
- (c) The volume  $V$ , found in part (b), changes as  $k$  changes. If  $\frac{dk}{dt} = \frac{1}{3}$ , determine  $\frac{dV}{dt}$  when  $k = \frac{1}{2}$ .

(a)  $f'(x) = 2e^{2x}$

$$\text{Perimeter} = 1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2} dx$$

3 :  $\begin{cases} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $\text{Volume} = \pi \int_0^k (e^{2x})^2 dx = \pi \int_0^k e^{4x} dx = \frac{\pi}{4} e^{4x} \Big|_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$

4 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c)  $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$

When  $k = \frac{1}{2}$ ,  $\frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$ .

2 :  $\begin{cases} 1 : \text{applies chain rule} \\ 1 : \text{answer} \end{cases}$

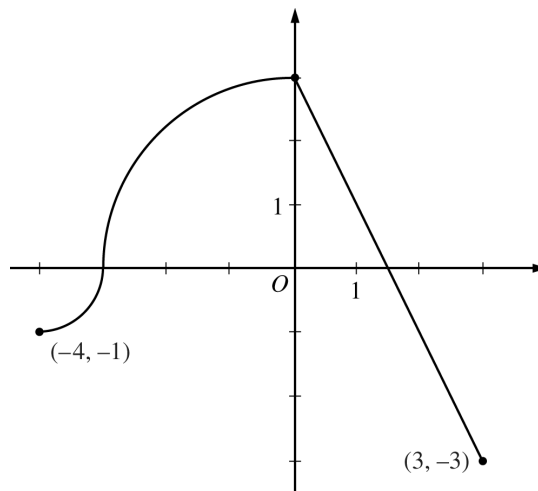
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**Question 4**

The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above.

Let  $g(x) = 2x + \int_0^x f(t) dt$ .

- (a) Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
- (b) Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
- (c) Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of  $f$

(a)  $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$   
 $g'(x) = 2 + f(x)$   
 $g'(-3) = 2 + f(-3) = 2$

$$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$$

(b)  $g'(x) = 0$  when  $f(x) = -2$ . This occurs at  $x = \frac{5}{2}$ .  
 $g'(x) > 0$  for  $-4 < x < \frac{5}{2}$  and  $g'(x) < 0$  for  $\frac{5}{2} < x < 3$ .  
 Therefore  $g$  has an absolute maximum at  $x = \frac{5}{2}$ .

$$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$$

(c)  $g''(x) = f'(x)$  changes sign only at  $x = 0$ . Thus the graph of  $g$  has a point of inflection at  $x = 0$ .

1 : answer with reason

(d) The average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$  is  
 $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$ .

$$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$$

To apply the Mean Value Theorem,  $f$  must be differentiable at each point in the interval  $-4 < x < 3$ . However,  $f$  is not differentiable at  $x = -3$  and  $x = 0$ .

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**Question 5**

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function  $W$  models the total amount of solid waste stored at the landfill. Planners estimate that  $W$  will satisfy the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  for the next 20 years.  $W$  is measured in tons, and  $t$  is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of  $W$  at  $t = 0$  to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time  $t = \frac{1}{4}$ ).
- (b) Find  $\frac{d^2W}{dt^2}$  in terms of  $W$ . Use  $\frac{d^2W}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time  $t = \frac{1}{4}$ .
- (c) Find the particular solution  $W = W(t)$  to the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  with initial condition  $W(0) = 1400$ .

(a)  $\left. \frac{dW}{dt} \right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is  $y = 1400 + 44t$ .

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411 \text{ tons}$$

(b)  $\frac{d^2W}{dt^2} = \frac{1}{25} \frac{dW}{dt} = \frac{1}{625}(W - 300)$  and  $W \geq 1400$

Therefore  $\frac{d^2W}{dt^2} > 0$  on the interval  $0 \leq t \leq \frac{1}{4}$ .

The answer in part (a) is an underestimate.

(c)  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$$\int \frac{1}{W - 300} dW = \int \frac{1}{25} dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$$

$$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$$

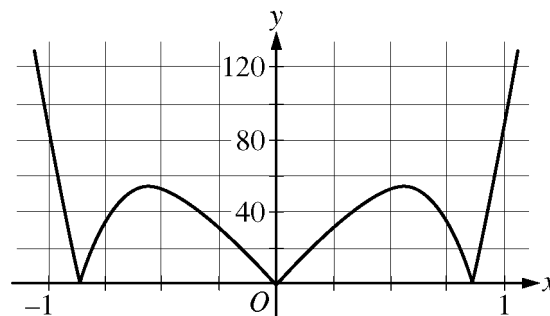
Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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**Question 6**

Let  $f(x) = \sin(x^2) + \cos x$ . The graph of  $y = |f^{(5)}(x)|$  is shown above.



Graph of  $y = |f^{(5)}(x)|$

- (a) Write the first four nonzero terms of the Taylor series for  $\sin x$  about  $x = 0$ , and write the first four nonzero terms of the Taylor series for  $\sin(x^2)$  about  $x = 0$ .
- (b) Write the first four nonzero terms of the Taylor series for  $\cos x$  about  $x = 0$ . Use this series and the series for  $\sin(x^2)$ , found in part (a), to write the first four nonzero terms of the Taylor series for  $f$  about  $x = 0$ .
- (c) Find the value of  $f^{(6)}(0)$ .
- (d) Let  $P_4(x)$  be the fourth-degree Taylor polynomial for  $f$  about  $x = 0$ . Using information from the graph of  $y = |f^{(5)}(x)|$  shown above, show that  $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$ .

(a)  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$   
 $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

3 :  $\begin{cases} 1 : \text{series for } \sin x \\ 2 : \text{series for } \sin(x^2) \end{cases}$

(b)  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$   
 $f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{121x^6}{6!} + \dots$

3 :  $\begin{cases} 1 : \text{series for } \cos x \\ 2 : \text{series for } f(x) \end{cases}$

(c)  $\frac{f^{(6)}(0)}{6!}$  is the coefficient of  $x^6$  in the Taylor series for  $f$  about  $x = 0$ . Therefore  $f^{(6)}(0) = -121$ .

1 : answer

(d) The graph of  $y = |f^{(5)}(x)|$  indicates that  $\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)| < 40$ .

2 :  $\begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$

Therefore

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \leq \frac{\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)|}{5!} \cdot \left(\frac{1}{4}\right)^5 < \frac{40}{120 \cdot 4^5} = \frac{1}{3072} < \frac{1}{3000}.$$